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Research Article

Approximate solution of space fractional diffusion equation

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Abstract

This paper is concerned with a new finite volume method using nodal basis functions for solving the space fractional diffusion equation with a space–time dependent variable coefficient subject to zero Dirichlet boundary conditions. The properties of nodal basis functions are exploited to reduce the space fractional diffusion equation to a system of ordinary differential equations that are then solved using the finite difference method. Some selected numerical simulations of space fractional diffusion equations are presented, and the results are compared with the exact solution and with the results obtained via other methods in the literature. The comparison reveals that the proposed method is reliable, effective, and accurate. All the computations were carried out using the Matlab package.

Keywords: Space fractional diffusion equation, Finite volume method, nodal function, zero Dirichlet boundary conditions

1. Introduction

Fractional partial differential equations in mathematical models have become increasingly popular recently. Different models using fractional partial differential equations have been suggested, and there has been important interest in developing numerical methods for their solution. Fractional partial differential equations can be grouped into two important types: space-fractional partial differential equations (SFPDEs) and time-fractional partial differential equations (TFPDEs).

Mathematical aspects of the boundary-value problems for the time-fractional diffusion equation and their applications in physics have been studied in Issa et al. (2023). Similarly, the space fractional diffusion equation is obtained by putting the second-order space derivative in the diffusion equation by an inverse Riesz potential of order $\beta > 0$ (Viguerie et al., 2021). The space-time fractional diffusion equation (SFDE), where the first-order time derivative is replaced with a Caputo-time fractional derivative of order $0 < \beta < 2$ and the author (Mainardi and

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Gorenflo, 1998) studied the second-order space derivative with a Riesz-Feller space-fractional derivative of order $0 < \alpha < 2$.

Several researchers have developed different numerical methods to solve space fractional diffusion equations. Among these, the author Ayalew (2024) studied Numerical solution of time fractional Klein-Gordon equation in framework of the Yang-Abdel-Cattani fractional derivative operator. Shifted Gegenbauer polynomials for solving space fractional diffusion equation was studied in Issa et al. (2023). Modified Legendre collocation method was studied in Jaleb and Adibi (2019) and shifted Gegenbauer-Gauss collocation method for solving fractional-differential equations with proportional delay was reported in Aghdam et al (2019). Different papers used orthogonal polynomials such as the four kinds of Chebyshev polynomials and their shifted forms for space fractional differential equations include (Aghdam et al., 2019; Sweila et al., 2016, Sweilam et al., 2015; Youssri et al., 2015). In this work, we consider the space fractional diffusion equation with a variable diffusion coefficient.

The structure of the remainder of this paper as follows. In Section 2, a new weighted fractional finite volume method for the space fractional diffusion equation with variable diffusion coefficient. In order to demonstrate the effectiveness of our theoretical analysis, some numerical results for the implicit fractional finite volume method are given in Section 3. Finally, the conclusions of the work are summarized in Section 4.

2. A new fractional finite volume method

In this section, we propose a new finite volume method using nodal basis functions for solving the space fractional diffusion equation with a space–time dependent variable coefficient subject to zero Dirichlet boundary conditions.

$$\frac{\partial u(x,t)}{\partial t} = \frac{\partial}{\partial x}\left[K(x,t)\left\{\beta \frac{\partial^{\alpha}}{\partial x^{\alpha}} + (1-\beta)\frac{\partial^{\alpha}}{\partial(-x)^{\alpha}}\right\}\right] u(x,t) + f(x,t) \tag{1}$$

We define $t_k = k\tau, k = 0,1,..,n, 0 < \alpha \leq 1$, let $\Omega = [a,b]$ be a finite domain, setting S_h be a uniform partition of Ω,which is given by $x_i = a + ih$ for $i = 0,1,...,m$,where $\tau = T/n$ and $h = (b-a)/m$ are the time and space steps, respectively. Firstly, we give the semi-discrete form of Eq. (1), the implicit Euler scheme:

$$\frac{u(x,t_k)-u(x,t_{k-1})}{\tau} - \frac{\partial}{\partial x}\left[K(x,t_k)\left\{\beta \frac{\partial^{\alpha}}{\partial x^{\alpha}} + (1-\beta)\frac{\partial^{\alpha}}{\partial(-x)^{\alpha}}\right\}\right] u(x,t_k) = f(x,t_k) \tag{2}$$

Furthermore, let $x_{j-1/2} = \frac{x_{j-1}+x_j}{2}, (j = 1,2,\ldots,m)$be the mid-point of the interval $[x_{j-1}, x_j]$then, formal integration of the governing equation over a control volume $[x_{j-1/2}, x_{j+1/2}]$for $j = 1,2,...,m-1$ gives

$$\begin{aligned} &\int_{x_{j-1/2}}^{x_{j+1/2}} u(x,t_k)dx - \tau\left[K(x,t_k)\left\{\beta \frac{\partial^{\alpha}u(x,t_k)}{\partial x^{\alpha}} + (1-\beta)\frac{\partial^{\alpha}u(x,t_k)}{\partial(-x)^{\alpha}}\right\}\right]\Bigg|_{x_{j-\frac{1}{2}}}^{x_{j+\frac{1}{2}}} \\ &\qquad = \int_{x_{j-1/2}}^{x_{j+1/2}} u(x,t_{k-1})dx + \int_{x_{j-1/2}}^{x_{j+1/2}} \tau f(x,t_k)dx \end{aligned} \tag{3}$$

let $S_h(a,b)$ denote the space of continuous and piecewise-linear functions with respect to the spatial partition, which vanish at the boundary $x=a$ and $x=b$ The nodal basis functions $\phi_i(x)$ for $i=1,2,...,m-1$ are of the following form:

$$\phi_i(x)=\begin{cases}\frac{x-x_{i-1}}{h} & x\in[x_{i-1},x_i],\\ \frac{x_{i+1}-x}{h} & x\in[x_i,x_{i+1}],\\ 0 & elsewhere.\end{cases} \tag{4}$$

And

$$\phi_0(x)=\begin{cases}\frac{x_1-x}{h} & x\in[x_0,x_1],\\ 0 & elsewhere.\end{cases} \tag{5}$$

$$\phi_m(x)=\begin{cases}\frac{x-x_{m-1}}{h} & x\in[x_{m-1},x_m],\\ 0 & \text{elsewhere}.\end{cases} \tag{6}$$

Then the approximate solution $u_h(x,t_k)\in p_h(a,b)$ with piecewise polynomials can be expressed as [11]

$$u_h(x,t_k)=\sum_{k=1}^{m-1}u_i^k\phi_i(x) \tag{7}$$

Substituting Eq. (7) in to Eq. (3), the corresponding finite volume scheme can be formulated as

$$\sum_{i=1}^{m-1}u_i^k\int_{x_{j-1/2}}^{x_{j+1/2}}\phi_i(x)dx-\tau\sum_{i=1}^{m-1}u_i^kK(x,t_k)\left\{\beta\frac{d^\alpha}{dx^\alpha}+(1-\beta)\frac{d^\alpha}{d(-x)^\alpha}\right\}\phi_i(x)\Big|_{x_{j-\frac{1}{2}}}^{x_{j+\frac{1}{2}}} \tag{8}$$
$$=\sum_{i=1}^{m-1}u_i^{k-1}\int_{x_{j-1/2}}^{x_{j+1/2}}\phi_i(x)dx+\tau\int_{x_{j-1/2}}^{x_{j+1/2}}f(x,t_k)dx$$

To solve Eq. (8), we recall the following results, which can be derived by direct calculation. For $i=1,2,...,m-1,$ we have

$$\int_{x_{j-1/2}}^{x_{j+1/2}}\phi_i(x)dx=\begin{cases}h/8, & when\ |j-i|=1\\ 3h/4 & when\ j=i\\ 0 & otherwise.\end{cases} \tag{9}$$

The operators $\frac{d^\alpha}{dx^\alpha}$ and $\frac{d^\alpha}{d\left(-x\right)^\alpha}$ are non-local operators. We can summarize the main results as follows:

$$\frac{d^\alpha}{dx^\alpha}\phi_i(x_{j+1/2})=\frac{1}{\Gamma(2-\alpha)h^\alpha}\begin{cases}0 & i>j+1\\ 2^{\alpha-1} & i=j+1\\ (^3-2^\alpha & i=j\\ c_{j-i+1} & i<j\end{cases} \tag{10}$$

$$\frac{d^\alpha}{dx^\alpha}\phi_i(x_{j-1/2})=\frac{1}{\Gamma(2-\alpha)h^\alpha}\begin{cases}0 & i>j\\ 2^{\alpha-1} & i=j\\ (^3-2^\alpha & i=j-1\\ c_{j-i} & i<j-1\end{cases} \tag{11}$$

$$\frac{d^{\alpha}}{d(-x)^{\alpha}}\phi_i(x_{j-1/2}) = \frac{1}{\Gamma(2-\alpha)h^{\alpha}}\begin{cases} c_{j-i+1} & i > j \\ (^3-2^{\alpha} & i = j \\ 2^{\alpha-1} & i = j-1 \\ 0 & i < j-1 \end{cases} \tag{12}$$

$$\frac{d^{\alpha}}{d(-x)^{\alpha}}\phi_i(x_{j+1/2}) = \frac{1}{\Gamma(2-\alpha)h^{\alpha}}\begin{cases} c_{j-i} & i > j+1 \\ (^3-2^{\alpha} & i = j+1 \\ 2^{\alpha-1} & i = j \\ 0 & i < j \end{cases} \tag{13}$$

Where $c_i = (i-3/2)^{1-\alpha} - 2(i-1/2)^{1-\alpha} + (i+1/2)^{1-\alpha}, i = 2,3,\ldots$ and $c_0 = 2^{\alpha-1}, c_1 = (3/2)^{1-\alpha} - 2^{\alpha}$,

$$\frac{h}{8}\left(u_{j-1}^{k} + 6u_j^{k} + u_{j+1}^{k}\right) - \sum_{i=1}^{m-1} u_i^{k} G_{ij} = \frac{h}{8}\left(u_{j-1}^{k-1} + 6u_j^{k-1} + u_{j+1}^{k-1}\right) + \tau\int_{x_{j-1/2}}^{x_{j+1/2}} f(x,t_k)dx \tag{14}$$

Where,

$$g_{1,ij} = \tau K(x_{j+1/2}, t_k)\left\{\beta\frac{d^{\alpha}\phi_i(x_{j+1/2})}{dx^{\alpha}} + (1-\beta)\frac{d^{\alpha}\phi_i(x_{j+1/2})}{d(-x)^{\alpha}}\right\} \tag{15}$$

$$g_{2,ij} = \tau K(x_{j-1/2}, t_k)\left\{\beta\frac{d^{\alpha}\phi_i(x_{j-1/2})}{dx^{\alpha}} + (1-\beta)\frac{d^{\alpha}\phi_i(x_{j-1/2})}{d(-x)^{\alpha}}\right\} \tag{16}$$

And

$$G_{ij} = g_{1,ij} - g_{2,ij} \tag{17}$$

Here, for $i = 1,2,\ldots,m-1$, we define

$$F_j^{k} = \int_{x_{j-1/2}}^{x_{j+1/2}} f_j^{k} dx \tag{18}$$

$$A_{ij} = \int_{x_{j-1/2}}^{x_{j+1/2}} \phi_j(x)dx, \quad U^{k} = [u_1^{n}, u_2^{n}, \ldots, u_{m-1}^{n}]^{T} \tag{19}$$

Thus, the Eq. (14) can be simplified as

$$\left(A_{ij} - G_{ij}\right)(U^{k})^{n} = A_{ij}(U^{k})^{n-1} + \tau\left(F_j^{k}\right)^{n} \tag{20}$$

Lemma 1, Assume that and $0 < \alpha < 1$, we define and c_i as

$$c_i = (i-3/2)^{1-\alpha} - 2(i-1/2)^{1-\alpha} + (i+1/2)^{1-\alpha}, i = 2,3,\ldots \tag{21}$$

Then c_i is increasing monotonically as i increases, and

$$c_i < 0, \ i = 2,3,\ldots \tag{22}$$

Proof. Setting function $f(x)$ and $F(x)$ in the interval $(0,+\infty)$ as

$$f(x) = x^{1-\alpha} - (x+1)^{1-\alpha}$$

And

$$F(x) = f(x) - f(x+1)$$

First, we shall prove $F(x) < 0$ for all $x > 0$. Since $x > 0$ and $0 < \alpha < 1$, then we have

$$f'(x) = (1-\alpha)[x^{-\alpha} - (x+1)^{-\alpha}] = (1-\alpha)\left[\frac{1}{x^{\alpha}} - \frac{1}{(x+1)^{\alpha}}\right] > 0$$

Therefore, $f(x)$ is increasing monotonically as x increases and $f(x) - f(x+1) < 0$, which means, for all $x > 0$, $F(x) < 0$. Now we consider the property of $F(x)$, since $x > 0$ and $0 < \alpha < 1$

$$f''(x) = -\alpha(1-\alpha)[x^{-\alpha-1} - (x+1)^{-\alpha-1}] = -\alpha(1-\alpha)\left[\frac{1}{x^{\alpha+1}} - \frac{1}{(x+1)^{\alpha+1}}\right] < 0$$

By using the Taylor expansion, we have

$$f(x+1) = f(x) + f'(x) + \frac{1}{2}f''(\xi) < f(x) + f'(x), \qquad \xi \in (x, x+1),$$

$$f(x-1) = f(x) - f'(x) + \frac{1}{2}f''(\eta) < f(x) - f'(x), \qquad \eta \in (x-1, x),$$

To sum each side of the inequality respectively, we obtain

$$f(x+1) + f(x-1) < 2f(x),$$

Which means

$$f(x-1) - f(x) < f(x) - f(x+1)$$

Namely,

$$F(x-1) < F(x).$$

Thus, for all $x > 0$, we have $F(x) < F(x+1)$. It is easy to check that $c_i = F\left(i - \frac{3}{2}\right)$, when $i \geq 2$, holds $c_i < 0$ and $c_i < c_{i+1.}$.

Hence, the proof is completed.

Lemma 2. Assume that $0 < \alpha < 1$, c_i is defined the same as (21), then

$$\lim_{i \to +\infty} c_i = 0 \tag{23}$$

And

$$\sum_{i=2}^{+\infty} \left(c_{i+1} - c_i\right) = -c_{2.} \tag{24}$$

Proof. By using the Taylor expansion, we get

$$\lim_{x \to +\infty} \left(x^\beta - 2(x+1)^\beta + (x+2)^\beta\right) = \lim_{x \to +\infty} x^\beta \left(1 - 2\left(1 + \frac{1}{x}\right)^\beta + \left(1 + \frac{2}{x}\right)^\beta\right)$$

$$= \lim_{x \to +\infty} x^\beta \left(1 - 2\left(1 + \frac{\beta}{x} + o\left(\frac{1}{x^2}\right)\right) + \left(1 + \frac{2\beta}{x} + o\left(\frac{1}{x^2}\right)\right)\right)$$

$$= \lim_{x \to +\infty} \left(1 + \frac{\beta}{x} + o\left(\frac{1}{x^2}\right)\right)$$

Where $0 < \beta < 1$.

Hence,

$$\lim_{i \to +\infty} \left((i - 3/2)^{1-\alpha} - 2(i - 1/2)^{1-\alpha} + (i + 1/2)^{1-\alpha}\right) = 0$$

Namely

$$\lim_{i \to +\infty} c_i = 0$$

To proof (24) can be obtained directly by summing $c_{i+1} - c_i$ from $i = 2$ to ∞ and apply (23), we get

$$\sum_{i=2}^{+\infty} \left(c_{i+1} - c_i\right) = c_3 - c_2 + c_4 - c_3 + c_5 - c_4 + \ldots = -c_{2.}$$

The L^2-norm error between the exact solution u^n and the numerical solution u_h^n is defined as [11]

$$E_2(h,\tau)=\sqrt{h\sum_{i=1}^{m-1}\left(u_i^n-u(x_i,t_n)\right)^2}$$

3. NUMERICAL RESULTS AND DISCUSSION

In order to demonstrate the effectiveness of our fractional finite volume method, the following numerical examples are presented.

Example: Liu et al. (2013) Consider the following space fractional diffusion equation with a variable coefficient

$$\frac{\partial u(x,t)}{\partial t}=\frac{\partial}{\partial x}\left[K(x,t)\left\{\beta\frac{\partial^\alpha}{\partial x^\alpha}+(1-\beta)\frac{\partial^\alpha}{\partial(-x)^\alpha}\right\}\right]u(x,t)+f(x,t) \tag{25}$$

$$u(x,0)=x^2(1-x)^2,\ 0\le x\le 1, \tag{26}$$

$$u(0,t)=u(1,t),\ 0\le t\le T, \tag{27}$$

$$K(x,t)=0.002(1+x(1-x)+t^2) \tag{28}$$

$$f(x,t)=e^t\begin{Bmatrix}x^2(1-x)^2-0.002(1-2x)[\beta p(x)+(1-\beta)p(1-x)]\\-K(x,t)[\beta q(x)-(1-\beta)q(1-x)]\end{Bmatrix} \tag{29}$$

$$p(x)=\frac{2x^{1-\alpha}}{\Gamma(3-\alpha)}\left[1-\frac{6}{3-\alpha}x\left(1-\frac{2x}{4-\alpha}\right)\right] \tag{30}$$

$$q(x)=\frac{2x^{1-\alpha}}{\Gamma(3-\alpha)}\left[1-\frac{6}{3-\alpha}x\left(1-\frac{2x}{4-\alpha}\right)\right]+\frac{12x^{2-\alpha}}{\Gamma(5-\alpha)}(\alpha-4+4x) \tag{31}$$

The exact solution is $u(x,t)=x^2(1-x)^2e^t$.

Define $r=\frac{\tau}{h^\alpha\Gamma(2-\alpha)}$ Therefore, we can rewrite Eq. (14) as

$$\begin{aligned}&\frac{h}{8}\left(u_{j-1}^k+6u_j^k+u_{j+1}^k\right)-r\frac{K_{j+1,k}+K_{j,k}}{2}\left[\beta\sum_{i=0}^{j}c_iu_{j+1-i}^k+(1-\beta)\sum_{i=0}^{m-1-j}c_iu_{j+i}^k\right]\\&+r\frac{K_{j,k}+K_{j-1,k}}{2}\left[\beta\sum_{i=0}^{j-1}c_iu_{j-i}^k+(1-\beta)\sum_{i=0}^{m-j}c_iu_{j-1+i}^k\right]\\&=\frac{h}{8}\left(u_{j-1}^{k-1}+6u_j^{k-1}+u_{j+1}^{k-1}\right)+\tau\int_{x_{j-1/2}}^{x_{j+1/2}}f(x,t_k)dx\end{aligned} \tag{32}$$

We take $\alpha=0.8,\beta=1,\tau=1/100=0.01$ and $h=(b-a)/m=1/8=0.125,\ j=1,2,...,m-1=8$

$$\begin{aligned}&\frac{h}{8}\left(u_{j-1}^k+6u_j^k+u_{j+1}^k\right)-r\frac{K_{j+1}^k+K_j^k}{2}\left[\sum_{i=0}^{j}c_iu_{j+1-i}^k\right]+r\frac{K_j^k+K_{j-1}^k}{2}\left[\sum_{i=0}^{j-1}c_iu_{j-i}^k\right]\\&=\frac{h}{8}\left(u_{j-1}^{k-1}+6u_j^{k-1}+u_{j+1}^{k-1}\right)+\tau F_j^k\end{aligned} \tag{33}$$

In Matrix form as follow

$$[A+B+C]U^k=AU^{k-1}+\tau F^k \tag{34}$$

Where:

$$U^k=\left[u_1^k,u_2^k,u_3^k,u_4^k,u_5^k,u_6^k,u_7^k\right]^T$$

$$U^{k-1}=\left[u_1^{k-1},u_2^{k-1},u_3^{k-1},u_4^{k-1},u_5^{k-1},u_6^{k-1},u_7^{k-1}\right]^T$$

$$F^k=\left[F_1^k,F_2^k,F_3^k,F_4^k,F_5^k,F_6^k,F_7^k\right]^T$$

The error between the exact solution u^n and the numerical solution u_h^n at $t=0.5,\alpha=0.8$ is

$$E_2(0.125.0.01)=9.36\times10^{-4}$$

Tabular expression of the approximate solution, exact solution and their comparison using $\alpha = 0.8$ and $t = 0.5$ (Table 1).

Table 1. Tabular expression of the approximate solution, exact solution and their comparison

X	u_{appro}	u_{exact}	$\lvert u_{exact} - u_{appro} \rvert$
0	0	0	0
0.125	2.0453e-02	1.9723e-02	7.30 e-04
0.25	5.9215e-02	5.7963e-02	1.25 e-03
0.375	9.1933e-02	9.0567e-02	1.37 e-03
0.5	1.0429e-01	1.0304e-01	1.25 e-03
0.625	9.1465e-02	9.0567e-02	8.98 e-04
0.75	5.8305e-02	5.7963e-02	3.42 e-04
0.875	1.8979e-02	1.9723e-02	7.40 e-04
1	0	0	0

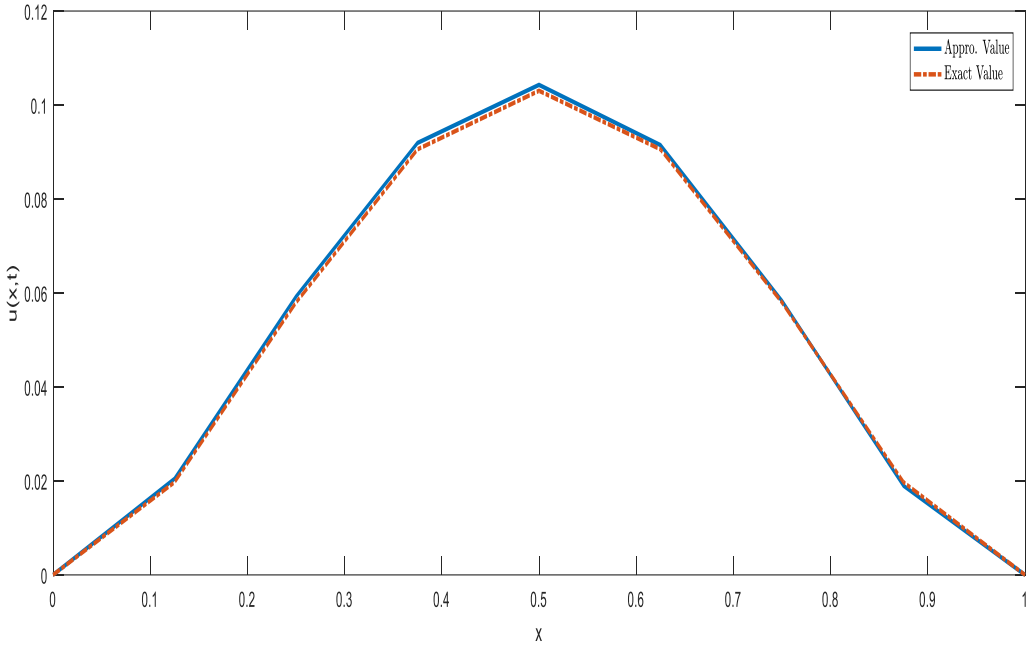


Figure 1. comparison of the numerical results with the analytical solution for $\alpha = 0.8$ and $t = 0.5$.

The error between the exact solution u^n and the numerical solution u_h^n at $t = 0.5, \alpha = 0.5$ is $E_2(0.125.0.01) = 9.42 \times 10^{-4}$

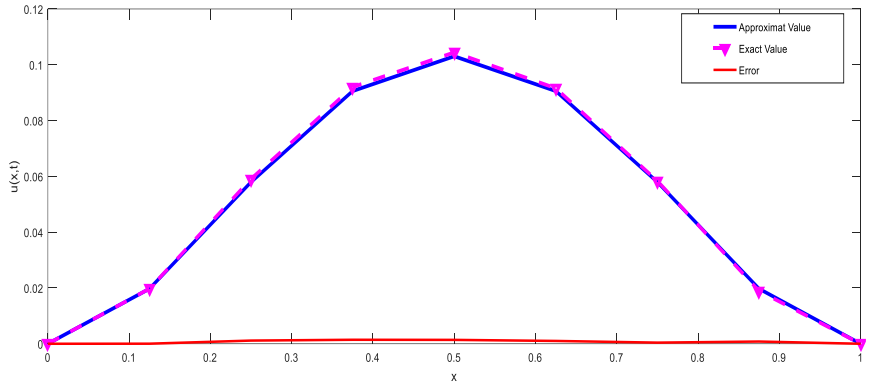


Figure 2. Comparison of the numerical results with the analytical solution for $\alpha = 0.5$ and $t = 0.5$.

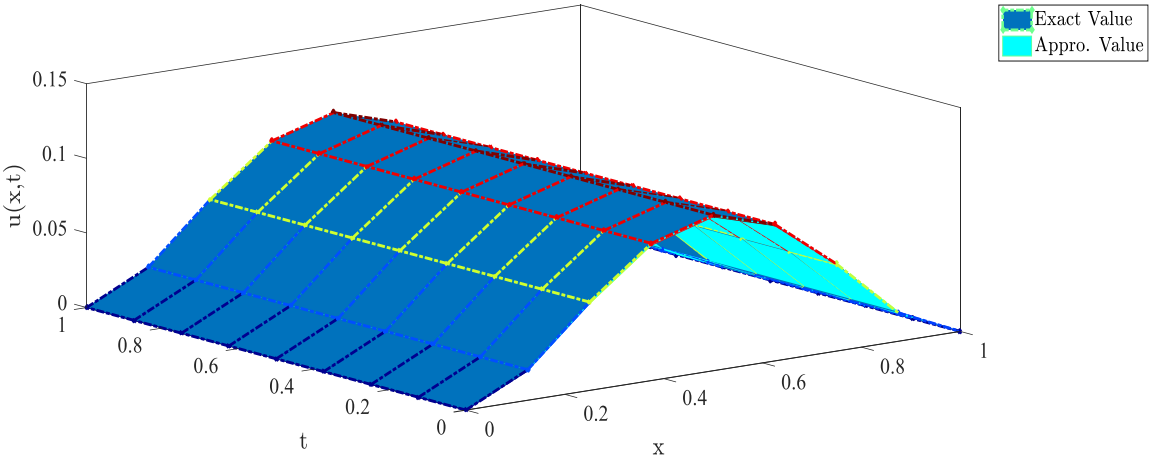


Figure 3. Mesh plot of the numerical results with the analytical solution of example 1 for different parameters α = 0.5 and t= 0.5.

The behavior of the solution at points in time $t = 0.5$ (second) When $\alpha = 0.8$ is displayed in Figure 1. An interesting feature is the accelerating of fractional diffusion equation solutions. In Eq. (25), using an order for the diffusion term (i.e. $\alpha = 0.5, \alpha = 0.8$) generates a rapidly decaying solution away from the center, shown in Figures 1 and 2. Even with an order of $1+\alpha$ near to 2, the solution takes up heavy tails when the diffusion term is replaced by a fractional one. The fractional diffusion equation solutions at $\alpha = 0.5$ at various points in time t (second) are shown in Figure 3. In this figure, the elongated tails in the x meter direction are clearly evident. The front is picking up speed, particularly along the coordinate axis. Dispersion kernels similar to those shown in Figure 2 are commonly shown in biological applications. This accelerating front could represent a forward-moving water flow. This distinguishing property of fractional diffusion equations could be useful in predicting the likely impacts of nuisance species

population management. The discrete numerical method solution still offers bounds on the solution of the continuous issue as the time and space steps approach zero, and the numerical method solution converges to the unique solution of the continuous problem.

4. CONCLUSION

To better illustrate our method, we have provided example. The numerical results as well as the numerical simulation show that the proposed method has great agreement with the exact solution. In addition, it is easy to observe that the accuracy of the method also depends on the fractional order of the partial differential equations.

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